

Midterm 1 - Review - Problems

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1 Limits

Problem 1

Find the following limits

(a) $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 - 9}$

(b) $\lim_{x \rightarrow 3^-} \frac{\ln(x)}{x-3}$

(c) $\lim_{x \rightarrow 0^+} \frac{(\ln(x))^2 + 1}{(\ln(x))^2 + 3}$

(d) $\lim_{x \rightarrow 0} \sin\left(\frac{\pi|x|}{x}\right)$

(e) $\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x}-\sqrt{3}}$

2 Epsilons and Deltas

Problem 2

Show that the following are true, using $\epsilon - \delta$

(a) $\lim_{x \rightarrow 1} x^2 - 2x = -1$

(b) $\lim_{x \rightarrow 1} x^2 + 2x = 3$

(c) $\lim_{x \rightarrow 1} x - \frac{1}{x} = 0$

3 Intermediate Value Theorem

Problem 3

Show that $2^x = x + 3$ has at least one solution in $(2, 3)$

4 Derivatives

Problem 4

Find the derivatives of the following functions, using the **definition** of the derivative:

(a) $f(x) = \frac{1}{x^2}$

(b) $f(x) = \sqrt{1 + 2x}$

Problem 5

Find the equation of the tangent line to the curve $y = x^4$ at $(2, 16)$ (you may use differentiation rules here!)

5 Functions-Stuff! (Chapter 1)

Problem 6

Find the domain of $f(x) = \sqrt{x^2 - 3x - 4}$

Problem 7

Find the inverse of $f(x) = 1 + e^{x^3}$

Problem 8

Sketch the graph of the following function and say at which points it is continuous:

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ \frac{1}{x} & \text{if } 1 < x < 3 \\ \frac{1}{2} + \sqrt{x-3} & \text{if } x \geq 3 \end{cases}$$